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Abstract: Common mode failure is a frequently occurred fault mode in the power system reliability evaluation. Two models of the common mode failure, named as 'individual model' and 'capacity model', are presented to overcome disadvantages of traditional models. Formularized reliability indices of each state and each capacity level can be evaluated quickly and accurately by using proposed models. Differences among these three models are also discussed. As a result, reliability indices calculated by two proposed models are correct and equal in theory, which are somewhat useful for the power system reliability evaluation.

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Common mode failure is a frequently occurred fault mode in power system reliability evaluation; it refers to multiple components outage simultaneously due to a common mode failure. Traditional model for a common mode failure is composed with independent outages into a composite state space model<sup>[1-5]</sup>. However, this model doesn't consider the situation where an independent outage and a common mode failure are occurred at the same time, meanwhile, the number of states increases exponentially with the increases of the number of components in this model, thus it will encounter "dimension disaster".

In this paper, it analyzes two weaknesses of traditional common mode models firstly, and then two new common mode models will be presented. Comparisons and discussions of the differences among these three models will be given out at the end of this paper.

#### Traditional model(the 1st model)

The traditional model for a common mode failure is to combine it with independent outages into a composite state space model. Such composite models have been widely used for years. Figure 1 shows an example of two components.

Applying the state space method to the composite model shown in figure 1, the probability of

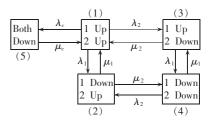


Fig.1 Composite model for common mode failure and independent outages

each state can be obtained as follows:  $p_1 = \mu_1 \mu_2 \mu_e / D_1, \ p_2 = \lambda_1 \mu_2 \mu_e / D_1, \ p_3 = \mu_1 \lambda_2 \mu_e / D_1$ (1)  $p_4 = \lambda_1 \lambda_2 \, \mu_c / D_1, \ p_5 = \mu_1 \, \mu_2 \lambda_c / D_1$ where  $D_1 = (\lambda_1 + \mu_1)(\lambda_2 + \mu_2) \mu_c + \mu_1 \mu_2 \lambda_c$ .

The traditional model has two evident weaknesses. The first, the composite model implies the assumption that an independent outage and a common mode failure are mutually exclusive. If figure 1 denotes two circuits on the same tower, for example, there is no state to represent a situation where an independent failure of the circuit(s) concurs with tower's failure. The second, when a common mode failure is associated with more than two components, the composite model becomes very complex. The number of states in the model increases exponentially with the number of components. This greatly increases difficulty in programming and calculations.

### Individual model(the 2nd model)

Because the event of common mode failure and

each independent outage can happen simultaneously, this state should be considered when modeling. A simple modeling approach is to use individual two-state models for the common mode failure and each independent outage and an intersection concept for combinations of them. The idea is straightforward and can be shown by figure 2.

$$Up \xrightarrow{\lambda_i} Down$$

All Up 
$$\mu_c$$
 All Down

(a) State space diagram of independent outages

(b) State space diagram of common mode failure

Fig.2 Individual model for common mode and independent outages

Probabilities of steady states of system illustrates in figure 2 can be expressed using the following equations<sup>[3]</sup>:

$$p_{iD} = \lambda_i / (\lambda_i + \mu_i) \qquad i = 1, \dots, n$$

$$p_{iU} = \mu_i / (\lambda_i + \mu_i) \qquad i = 1, \dots, n$$

$$p_{eD} = \lambda_e / (\lambda_e + \mu_e), \quad p_{eU} = \mu_e / (\lambda_e + \mu_e)$$
(2)

The subscript i indicates the ith component and there are n components in total. The subscript c indicates the common mode failure of the components. The  $\lambda$  (failures/year) and  $\mu$  (repairs/year) are failure and repair rates. The  $p_{i\mathrm{D}}$  and  $p_{i\mathrm{U}}$  are probabilities of the ith component in the down and up states. The  $p_{c\mathrm{D}}$  and  $p_{c\mathrm{U}}$  are probabilities of the common mode failure occurring and not occurring, respectively.

It can be seen from equation (2) that the equations for independent outages and common mode failures are the same in the mathematical form. However, they have definitely different meanings—when an independent outage takes place, only one component fails while when the common mode failure happens, all n components fail.

In using the individual models, a combined state is defined as a combination of common mode and independent outages and its probability is calculated by equation (3).

$$p_{j} = \sum_{i \in s_{a}} p_{i \cup i} \sum_{i \in s_{a}} p_{i \cup i} \max \{ r \, p_{e \cup i}, (1 - r) \, p_{e \cup i} \}$$
 (3)

Where  $p_j$  denotes the probability of any combined state j,  $s_u$  and  $s_d$  are the sets of components whose independent outage does not happen and happens in state j, respectively, and r is a variable having only two values of 0 or 1, with 0 indicating that the common mode failure does not happen in state j and 1 otherwise.

All combined states of system state space can be calculated by using equation (3), and the result is as follows:

$$p_{1} = \mu_{1} \mu_{2} \mu_{e} / D_{2}, p_{2} = \lambda_{1} \mu_{2} \mu_{e} / D_{2}, p_{3} = \mu_{1} \lambda_{2} \mu_{e} / D_{2}$$

$$p_{4} = \lambda_{1} \lambda_{2} \mu_{e} / D_{2}, p_{5} = \mu_{1} \mu_{2} \lambda_{e} / D_{2}$$

$$p_{0} = (\lambda_{1} \lambda_{2} + \mu_{1} \lambda_{2} + \mu_{2} \lambda_{1}) \lambda_{e} / D_{2}$$

$$(4)$$

Where  $D_2 = (\lambda_1 + \mu_1)(\lambda_2 + \mu_2)\mu_c + \mu_1\mu_2\lambda_c + (\lambda_1\lambda_2 + \mu_1\lambda_2 + \mu_2\lambda_1)\lambda_c$ .

This model overcomes the two weaknesses of the first model in both aspects. Firstly, any events including ones of concurrence of any independent outage and the common mode failure can be modeled using a combination concept. Secondly, any number of components in a common mode failure can be easily and directly considered by using an enumeration technique.

## 3 Capacity model(the 3rd model)

This section presents a novel common mode model named as capacity model, which can be used to analyzing the effect of common mode failures on transmission capability of a system. The probabilities of system states which descending sorted as capacity amount can be calculated by using the model.

The capacity model is illustrated using two circuits on the same tower; tower collapse is common mode failure to the two circuits. So, common mode failure can be treated as a virtual component, which will be in series with the parallel combination of other individual components. The capacity of the virtual component will be defined as the sum of individual components capacity. Figure 3 illustrates this reliability structure.



Fig.3 Reliability structure of common mode failure

So, reliability evaluation of a system with common mode failure can firstly analyzing reliability model of each component, then combine them in parallel and in series. In this section, capacity model of each component and each system is illustrated in table 1<sup>[6]</sup>.

Tab.1 Basic form of capacity model

Capacity	Probability	Cumulative probability
$x_1$	$p_{1}$	$P_{1}$
$x_2$	$p_{2}$	$P_2$
÷	:	÷
$\mathcal{X}_n$	$p_{_n}$	$P_{_{n}}$

Where,  $x_i$  denotes the capacity of state i,  $x_1$  and  $x_n$  are full and zero capacity respectively,  $p_i$  (lowercase) is the corresponding probability of state i;  $P_i$  (capital, is the cumulative probability of state i) is the summation of probabilities of all state whose capacity is r equal to or less than the capacity of state i.

There has many descending capacity states in capacity model, as far as one transmission line is concerned, the capacity states are 1 (full capacity in

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p.u.) or 0(zero capacity in p.u.), while for a two -circuit system, capacity states of the system is 1, 0.5 and 0(p.u.).

Figure 3 denotes that individual components and common mode component are in series connection in reliability structure. In regard to series system, cumulative probability of capacity X can be calculated by using equation  $(5)^{[6]}$ .

$$P(X) = \sum_{i} P_1(X - b_i) [P_2(b_i) - P_2(b_{i+1})]$$
 (5)

Where, j denotes the system state number of one of the series connection systems;  $b_j$  is the capacity amount corresponding to state j. As an example, in figure 3, j represents 1 or 2, because common mode model has two capacity model, i.e. 1 capacity  $(b_1, \text{in state 1})$  and 0 capacity  $(b_2, \text{in state 2})$ .

Individual components are in parallel in reliability structure, cumulative probability of capacity X of parallel system can be calculated by using equation  $(6)^{[6]}$ .

$$P(X) = P_1(X) + P_2(X) - P_1(X)P_2(X)$$
 (6)

The system states of this model are identical with the 2nd model. If the full capacity of component 1 and 2 is 1 (p.u.), then the full capacity of common mode component is 2(1+1=2). So, the system illustrates in figure 3 contains three capacity states, namely 2,1 and 0, cumulative probabilities of these three states can be calculated by using equation (5) and (6), the result is shown in equation (7).

$$P(2) = 1$$

$$P(1) = p_{1D} + p_{2D} - p_{1D} p_{2D} + p_{cD} - p_{cD} (p_{1D} + p_{2D} - p_{1D} p_{2D}) = [(\mu_1 \mu_2 + \lambda_1 \lambda_2 + \mu_1 \lambda_2 + \lambda_1 \mu_2) \lambda_c + (\lambda_1 \lambda_2 + \lambda_1 \mu_2 + \mu_1 \lambda_2) \mu_c] / D_3$$

$$P(0) = p_{1D} p_{2D} + p_{cD} - p_{1D} p_{2D} p_{cD} = (\lambda_1 \lambda_2 \lambda_c + \mu_1 \lambda_2 \lambda_c + \mu_2 \lambda_1 \lambda_c + \mu_1 \mu_2 \lambda_c + \lambda_1 \lambda_2 \mu_c) / D_3$$

$$P(0) = p_{1D} p_{2D} + p_{2D} + p_{2D} p_{2D} p_{cD} = (\lambda_1 \lambda_2 \lambda_c + \mu_1 \lambda_2 \lambda_c + \mu_2 \lambda_1 \lambda_c + \mu_2 \lambda_1 \lambda_c + \mu_1 \mu_2 \lambda_c + \lambda_1 \lambda_2 \mu_c) / D_3$$

Where  $D_3 = (\lambda_1 + \mu_1)(\lambda_2 + \mu_2)\mu_c + \mu_1 \mu_2 \lambda_c + (\lambda_1 \lambda_2 + \mu_1 \lambda_2 + \mu_2 \lambda_1)\lambda_c$ .

# 4 Comparisons among the three models

It can be seen that there is one more term in the denominator of equation (4) than in equation (1), which is  $(\lambda_1\lambda_2 + \mu_1\lambda_2 + \mu_2\lambda_1)\lambda_c$ . However, since  $\lambda << \mu$ , this term is extremely smaller compared to the remaining portion in the denominator. Thus this term can be neglected. So the probability of concurrence of independent and common mode failure  $p_0$  is extremely low. This is due to the assumption that an independent outage and a common mode failure are mutually exclusive in the 1st mode, but it is possible existence from practice point of view.

The 2nd model takes all-possible system states into consideration, thus overcome the weakness of the 1st model, and is better than the 1st model in modeling and calculation. But unfortunately, the 2nd model is only convenient for the calculation of each system states; the calculation will be cumbersome and fallible if the needed reliability indices are related to system transmission capability, such as loss of load probabilities.

The 3rd model is based on capacity amount, it regards common mode failure as a component failure, thus switch the analysis of common mode to the analysis of series and parallel system, it is more clear and precise than the 1st and 2nd model. The reliability based system transmission capability becomes very easy through conjoining series and parallel formulas of capacity model, thus overcomes the weakness of the 2nd model. However, it is better to use the 2nd model if the reliability requirement is probability of individual states.

If classify system states of the 2nd model according to capacity, the 2(p.u.) capacity state is  $p_1$  of equation (4), the 1(p.u.) capacity state is the sum of  $p_2$  and  $p_3$ , the 0(p.u.) capacity state is the sum of  $p_0, p_4$  and  $p_5$ . The result is that cumulative probability of 0 capacity is equal to P(0) of equation (7), cumulative probability of 1 capacity is equal to P(1) of equation (7), cumulative probability of 2 capacity is equal to P(2) of equation (7). On the contrary, equation (4) can be derived if decompose equation (7). So the 2nd model and the 3rd model are equal in mathematic form.

#### 5 Example results

As an example, two circuits on the same tower are taken into consideration. Circuit 1 is indicated with subscript as 1; by the same token, circuit 2 as 2, common mode failure is indicated as c.

Lets.

$$\lambda_1 = 0.1 (f/y),$$
  $\mu_1 = 1000 (f/y)$   
 $\lambda_2 = 0.2 (f/y),$   $\mu_2 = 1000 (f/y)$   
 $\lambda_c = 0.01 (f/y),$   $\mu_c = 3000 (f/y)$ 

Assume that each circuit has the transmission capability of 1(p.u.), so the transmission capability of common mode failure is 2. The probabilities of all states of model 1st and model 2nd have listed as follows:

$$\begin{split} &D_{\rm 1st} \!=\! 3.000\ 910\ 06\times 10^9\\ &D_{\rm 2nd} \!=\! 3.000\ 910\ 063\ 000\ 2\times 10^9\\ &p_{\rm 1.1st} \!=\! 9.996\ 967\ 386\ 620\ 044\times 10^{-1}\\ &p_{\rm 1.2nd} \!=\! 9.996\ 967\ 376\ 625\ 443\times 10^{-1}\\ &p_{\rm 2.1st} \!=\! 9.996\ 967\ 386\ 620\ 045\times 10^{-5} \end{split}$$

 $p_{2.2nd}$ =9.996 967 376 625 442 ×10<sup>-5</sup>  $p_{3.1st} = 1.999393477324009 \times 10^{-4}$  $p_{3,2\text{nd}}$  = 1.999 393 475 325 089  $\times 10^{-4}$  $p_{4.1st} = 1.999393477324009 \times 10^{-8}$  $p_{4.2 \text{nd}} = 1.999 \ 393 \ 475 \ 325 \ 089 \times 10^{-8}$  $p_{5.1st} = 3.332322462206681 \times 10^{-6}$  $p_{5.2nd}$  = 3.332 322 458 875 147 × 10<sup>-6</sup>  $p_{0,2\text{nd}}$ =9.997 633 841 117 217  $\times 10^{-10}$ 

It can be seen that probability of the same state from 1st model and 2nd model has only a little difference because of the consideration of the where an independent outage and a common mode failure are occurred at the same time, the probability is  $p_0$ . It can also be noted that  $p_0$  is very small relative to  $p_1 : p_5$ , so the result from 1st model can also be used, and it has been widely used for years<sup>[1-5]</sup>.

Probabilities of each capacity states from model 3rd is as follows:

> $D_3 = D_2$ , P(2) = 1 $P(1) = 3.032623374557758 \times 10^{-4}$  $P(0) = 3.353 316 157 012 511 \times 10^{-6}$

From 2nd model, it is clear that states of transmission capacity no more than 0 is state 0, state 4 and state 5, the sum of probability of these states  $(p_0 + p_4 + p_5)$  is equal to P(0), transmission capability less than or equal to 1 is all the states except state 1, the probability is equal to P(1).

So, the result from the 2nd mode and the 3rd model is equal in value, the 2nd model can deal with each probable state, and the 3rd model can dispose of the reliability base on transmission capability, so, the virtues of each model are complementary and can be used in different calculation.

#### Conclusions

The effect of common mode failure on reliability indices should be considered in power system reliability evaluation. In this paper, it focuses on common mode model, by analyzing two weaknesses of traditional common mode model; two new common mode models are presented.

The three studied models have different characteristics, and can be used in different reliability objective. In the 1st model, the transfer process of system state is demonstrated by state space, it is easy to understand, and the error is acceptable; in the 2nd model, probabilities of each state can be draw out accurately, but the system states and their transfer processes is blind, system state space should

be appreciated firstly, and the result is probability of each state, it can't realize the effect of common mode failure on system transfer capability; the 3rd model is based on capacity model, probability of each capacity state is formularized, it is easy to calculate other reliability indices, such as availability, loss of load probability, and so on, and this model can be used to deal with common mode failure of a system with descending capacity state.

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# 电力系统可靠性评估的共模故障模型研究

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摘要:共模故障是电力系统可靠性评估中常见的一种故障模式。针对传统共模故障模型的不足,提出了2种新的共模故障模型,即独立模型和容量模型。利用这2种模型,可分别公式化求得系统各个状态和各个容量水平的可靠性指标,结果准确,求解速度快。最后,对3种模型各自的优缺点进行分析和比较,由所提2种模型求得的可靠性指标是准确的,两者在理论上也是等价的,所得结果对电力系统可靠性评估有一定的指导意义。

关键词: 电力系统; 可靠性评估; 共模故障: 模型

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